OUR CURRICULUM

MATHEMATICS

PHILOSOPHY & NARRATIVE

Beauty is the first test; there is no permanent place in the world for ugly mathematics.

G. H. Hardy

OUR PHILOSOPHY

Mathematics is the language of the universe; understanding the wonder of mathematics is to experience a life in full colour.

The study of mathematics starts with modelling the concrete world around us, defining number and investigating patterns.

These patterns lead us to abstract generalisations and rules for which a structure of logic and proof is required. Further study leads us to discoveries about the world that we have never considered before.

Scientists at the fore-front of research are using mathematical models to make predictions about the composition of the universe and life itself.

Too often, mathematics is seen, and taught, as a utilitarian tool, a servant of the sciences.

For those starting the journey into the world of mathematics, it is hard to see the wonders ahead, and time has to be given to mastering the basics and developing fluency.

Only when the studies become more advanced, can the true beauty of mathematics be seen. The unexpected elegance and simplicity of the results that we find resonate with the aesthetic. Those who can appreciate such beauty have their eyes opened to wonders which others cannot even begin to comprehend.

KEY STAGE THREE

We begin by developing mathematical fluency through learning the fundamentals of number and calculation, developing a confident recognition, understanding and manipulation of rational numbers through a variety of approaches and representations.

As students' mastery of number begins to develop, we move on to abstract generalisation with the introduction of the grammar of algebra, alongside the introduction of the grammar of formal geometry and statistics.

The final step of a mathematician is to create rigorous proof. The emphasis of the curriculum is on developing a numerical confidence which comes from practising the foundations of the discipline. Throughout their journey, students only move onto the next stage when they have achieved a level of competence within the mastery curriculum. For most students, the journey starts with the representation of number in decimal, percentage and fractional forms, and being able to calculate and manipulate them fluently. We consider the growth of the number line, from counting numbers and rational numbers, to the introduction of zero and negative numbers in 7th century India.

We look into factors and multiples of a number, and consider one of the oldest pastimes of number theory: prime numbers. Touching on Euclid's proof, we gain an insight into how prime numbers have become the core of internet security. We develop logic and strategic problem-solving by identifying patterns and recognising proportional relationships.

The study of geometry begins with construction and measurement using the Babylonian angle measure of degrees. We develop students' concrete understanding before later treating shapes in a more abstract way, and introducing calculations using basic angle facts of shapes. We take a similar approach with measuring area and volume, eventually starting to generalise results with formulae.

We introduce the students to statistics, with calculations derived from simple data representations such as bar charts, pictograms and line graphs. This extends to average calculations and simple analysis of the data produced. As we move on from numerical calculations, we start to try to generalise results with the introduction of the grammar of algebra. This is really the bedrock of the subject for those who will eventually go on to study it at a higher level where fluency is a must. Solving equations, collecting like terms and factorising are all fundamental skills required.

These skills form a web, linking topic areas together, for example looking at angles in a polygon with algebraic values and average calculations with values missing. We build on the grammar, promoting conceptual understanding in the form of rhetoric, allowing students to choose the best method to solve a problem.

KEY STAGE FOUR

Students begin to excel in the different areas of mathematics: fluency in calculations; spotting patterns in number, and being able to formalise generalised relationships; manipulation of complex algebra; geometric reasoning and analysis of statistical measures.

We concentrate on the foundations of formal proof, creating logical and rigorous arguments: essential rhetoric for a mathematician. Students are taught to approach unseen problems with a range of strategies, and use mathematical insight to identify the most efficient. The lines between areas of mathematics start to blur as we begin to appreciate the inter-relatedness of the different disciplines.

We start with the most basic of polygons, the triangle, and learn of Pythagoras and the theorem named after him. This leads naturally on to trigonometric calculations for right-angled triangles and then onto generalised rules for all triangles. We return to Euclid and his work looking at geometry, building up from angles in triangles to generalised polygons and then onto proofs of circle theorems.

This area of mathematics is based firmly in Ancient Greece, where only straight edges and compasses were available to make constructions and loci, which leads us to the quest to find the area of a circle, from a Biblical reference to Archimedes' method of exhaustion. Archimedes then leads us to the volume of prisms and density calculations.

Statistics remains firmly grounded in calculations, extending the range of diagrams, and adding in more advanced analysis. Students are introduced to probability calculations and Venn diagrams leading to an understanding of basic conditional probability.

Work on the number line leads us to an introduction to surds and irrational numbers, which completes the set of real numbers. We look at standard form linking in with calculations in Science which leads to compound measures such as speed and pressure. Building on our earlier work on proportional relationships, students generalise direct and inverse proportion.

Algebra takes prominence at this level of study. Starting with Descartes' graphical representation of algebraic functions, we analyse the fundamental link between solutions and graphs of equations and this develops further to generalised graph transformations. The graph work is completed with some pre-calculus calculations of rates of change.

Building on previous work, we solve quadratic and simultaneous equations using a host of different methods and understanding the advantages of each. We look at inequalities and equations which cannot be solved exactly, and thus extending students' repertoire to include iteration: a powerful tool used by computers. We link back to concrete examples developing more and more sophisticated models for real life situations such as exponential growth and decay.

The final part of the journey is to develop the rhetoric of algebraic proof, making logical and rigorous arguments and paving the way for further study.

KEY STAGE FIVE

We start by deepening the knowledge of the algebraic concepts learned previously: learning to rationalise the denominator; proving that completing the square and the quadratic formula are one and the same; expanding our knowledge of polynomials to cubics and quartics; seeing Euclidean geometry and Cartesian graphs merge.

Proof becomes a topic in its own right as we look at different methods: by exhaustion; by deduction; by contradiction. We take a look at Pascal's triangle, and how we can generalise with factorial notation leading to the binomial expansion.

We study arithmetic and geometric sequences and learn of Gauss' schoolboy trick for summing. Our study of trigonometry introduces radian measure and calculation, as well as looking at trigonometric identities and proofs of double angle formulae. Work on exponential relationships is extended by the learning of Napier's logarithms.

The completely new topic of calculus is introduced as we learn of Newton and Leibniz's concurrent discoveries, allowing us to differentiate and integrate. We expand our repertoire of techniques to cover the differentiation of almost all functions.

We also look at numerical methods such as the trapezium rule and the Newton-Raphson method. We change from Cartesian to parametric form.

The applied area of mechanics puts context to the pure side of the course, allowing for calculations between displacement, velocity and acceleration, bringing in calculus, and applying vector notation. We study Newton's laws of forces and consider projectiles and moments.

In Statistics, we look at the whole data collection cycle: sampling, calculation, representation and analysis. In our study of probability, we build on prior knowledge to start looking at particular distributions: binomial and Gauss' Normal distribution, this leads to hypothesis testing to aid in our analysis of the statistics by understanding the significance of results.

BUILDING ON THE KNOWLEDGE OF THE PAST
TO HELP THE CHILDREN OF TODAY
MEET THE CHALLENGES OF TOMORROW