USING NUMBER...

Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent
Negative: A value below zero.
Coefficient: The number used to multiply a variable

Square and cube numbers

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Square numbers

1, 4, 9, 16...

144 = 2 x 2 x 2 x 2 x 3 x 3

(2 x 2 x 3) x (2 x 2 x 3)

12 x 12

Prime factors can find square roots

$\sqrt{144} = 12$

Cube numbers

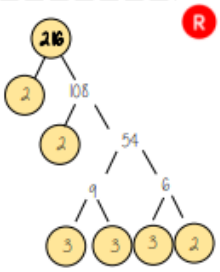
1, 8, 27, 64, 125...

216 = 2 x 2 x 2 x 3 x 3 x 3

(2 x 3) x (2 x 3) x (2 x 3)

6 x 6 x 6

$\sqrt[3]{216} = 6$



Higher powers and roots

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$

$= 9 \times 100$

$= 900$

Standard form

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Any integer

Any number between 1 and less than 10

$A \times 10^n$

0.001
1 x $\frac{1}{1000}$
1 x 10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Example

3.2 x 10^4
= 3.2 x 10 x 10 x 10 x 10
= 32000

Non-example

(0.8) x 10^4

5.3 x $10^{(0.7)}$

Numbers in standard form with negative powers will be less than 1

3.2 x 10^{-4} = $3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$ = 0.00032

Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$(x^a)^b = x^{ab}$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated. Use the addition law for indices

$(2^3)^4 = 2^{12}$ ← $a \times b = 3 \times 4 = 12$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

Standard form calculations

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Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

6 x 10^5 + 8 x 10^5

Method 1

= 600000 + 800000
= 1400000
= **1.4 x 10^6**

This is not the final answer

Method 2

= (6 + 8) x 10^5
= 14 x 10^5
= 1.4 x 10^1 x 10^5
= **1.4 x 10^6**

Multiplication and division

1.5 x 10^5
0.3 x 10^3

Division questions can look like this

(1.5 x 10^5) ÷ (0.3 x 10^3)

(1.5 ÷ 0.3) x $10^5 \div 10^3$

= **5 x 10^2**

For multiplication and division you can look at the values for **h** and the powers of 10 as two separate calculations